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TWO PHASE NATURAL CONVECTION ADJACENT TO A VERTICAL HEATED SURFACE IN A PERMEABLE MEDIUM

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Abstract—Two phase natural convection adjacent to a heated, vertical surface in a permeable medium is treated using boundary-layer approximations for conditions under which the vapor layer adjacent to the heated surface is thin. A similarity solution is obtained for the case in which density differences due to the phase change dominate those due to temperature variations in the liquid phase outside the vapor layer and for which the surface temperature, the temperature far from the surface, and the heat of vaporization do not vary with distance along the surface. The application of this solution to problems of practical interest using the approximation of local similarity is discussed.

NOMENCLATURE

c_p, specific heat of fluid;
g, acceleration of gravity;
H, heat of vaporization per unit mass;
k_m, thermal conductivity of permeable medium;
K, permeability;
l, physical length scale defined in equation (28);
Nu_z, local Nusselt number;
p, fluid pressure;
q, local heat-transfer rate;
Ra_z, local Rayleigh number;
T, temperature;
u_x, horizontal velocity;
u_z, vertical velocity;
x, horizontal coordinate;
z, vertical coordinate.

Greek symbols

α, parameter appearing in similarity solution of equation (26);
β, coefficient of thermal expansion;
δ, vapor layer thickness;
ν, kinematic viscosity;
ρ, fluid density.

Subscripts

L, liquid phase;
V, vapor phase;
0, value on vertical surface $x = 0$;
Δ, value on phase change boundary;
∞, value at large x.

INTRODUCTION

THE STUDY of natural convection in permeable media has important applications in both engineering and the earth sciences. One important geological application is to the cooling of bodies of igneous rock emplaced into permeable crustal rock near the surface of the earth. In areas of abundant igneous activity, the crystallization and cooling of igneous intrusions represents a significant source of geothermal energy.

Early studies of the cooling of igneous bodies considered heat conduction to be the principal mechanism of heat transfer from an intrusion to cooler, surrounding rock [1–3]. However, presently active geothermal systems provide abundant evidence for the movement of heated groundwater in some areas of igneous intrusion, and it is generally accepted that groundwater movement is an important mechanism of heat transfer in such areas (c.f. [4]). More recently, geochemical studies of igneous rocks [5] demonstrate that groundwater has moved through rock both in and around a number of intrusions. There is thus the need to understand natural convection of groundwater in permeable rock surrounding igneous intrusions.

Thermal convection has been long recognized as a simple mechanism for generating groundwater motion and a variety of studies of thermal convection in permeable media have been reported [6–9]. The most detailed studies of the effect of groundwater circulation on the cooling of igneous intrusions thus far reported are those of Cathles [10] and Norton and Knight [11]. In these studies finite difference models of cooling intrusions were developed and were applied to investigate the effect of physical parameters such as the size, depth and permeability of an intrusion.

This study treats the problem of flow in a permeable medium adjacent to a heated, impermeable vertical surface. An earlier study by Cheng and Minkowycz [12] has considered this problem for flow of a single phase liquid. The present study considers the natural convection of a fluid which vaporizes in the vicinity of the heated surface.

In many geological situations it is reasonable to suppose that circulating groundwater within permeable rock surrounding an intrusion is at a pressure close to hydrostatic which increases by about one-tenth of a kilobar (10 MPa) for each kilometer of depth below the water table. The critical pressure of pure water is about 220 bars (22 MPa) corresponding to a depth of slightly more than 2 km. Saline water can have significantly higher critical pressure [13] and therefore be subcritical to greater depths. Some types of igneous intrusions are emplaced at temperatures in excess of 1000°C into rock at temperatures of several hundred °C. At the time of emplacement, the temperature at the contact between intrusive and surrounding rock will be approximately the average of the intrusion temperature and that of the surrounding rock. Therefore contact temperatures may be initially as high as 600°C, and, for pressures less than the critical pressure, water near the contact will be in the vapor phase during the early stages of the cooling of such an intrusion. For purely conductive heat transfer, Jaeger [3] has considered the latent heat effect of vaporizing groundwater contained in porous rock surrounding an intrusion. The finite difference models of Cathles [10] and Norton and Knight [11] also treat vaporization by accounting for enthalpy variations due to phase changes and by using an empirical equation of state for water. These studies do not, however, treat the flow of mixed phases. A study by Donaldson [14] treats the problem of mixed phases in the case of a horizontally uniform, vertical flow.

In this paper natural convection due to the formation of a vapor layer adjacent to a heated vertical surface in a permeable medium is treated using boundary-layer approximations with the assumption that the vapor layer is thin. Boundary-layer approximations have been previously applied to thermal convection of a single phase fluid in a permeable medium [12, 15–18]. Here the boundary-layer formulation for a vertical vapor layer is developed, and a similarity solution is obtained for the case in which density differences due to the phase change dominate those due to temperature variations in the liquid phase outside the vapor layer. The treatment of this problem is greatly simplified by the fact that no region of mixed phases occurs. Application of these results to studies of the cooling of igneous intrusions using a local similarity approximation is discussed.

FORMULATION OF BOUNDARY-LAYER APPROXIMATIONS

To develop the boundary-layer approximations for a vapor layer in a permeable medium adjacent to a heated vertical surface, the (x, z) coordinate system shown in Fig. 1 is introduced. The vapor layer is taken to be of thickness δ which increases with increasing z. Boundary-layer approximations are introduced with the assumption that the vapor layer is thin. Since there is no geometrical length scale with which to compare the thickness, a thin layer is

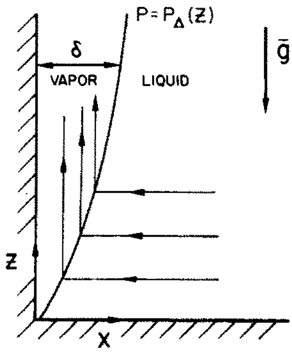

FIG. 1. Coordinates and nomenclature for a thin vapor layer adjacent to a heated vertical surface in a permeable medium.

assumed to be one for which $\varepsilon = d\delta/dz$ is small compared to unity.

Flow in a permeable medium is described by the equation for conservation of mass and by Darcy's law. For steady state flow these are given by

$$\nabla \cdot \rho \bar{u} = 0 \tag{1}$$

and

$$\frac{\nu}{K} \rho \bar{u} = -\nabla p + \rho \bar{g} \tag{2}$$

respectively. Here $\bar{u}$ is the Darcian fluid velocity, ρ is the fluid density, p is the pressure, $\bar{g}$ is the acceleration of gravity, ν is the kinematic viscosity of the fluid, and K is the permeability of the medium. If the fluid density field is known or can be calculated from a suitable equation of state, equations (1) and (2) determine the pressure and velocity fields.

First consider the flow within the vapor layer $x < \delta$. The density of the vapor phase is assumed to be small compared to that of the liquid phase so that the body force term which appears in equation (2) and which represents the hydrostatic contribution to the vertical pressure gradient may be neglected within the vapor layer. Darcy's law then reduces to

$$\nabla p = -\frac{\nu}{K} \rho \bar{u}. \tag{3}$$

For constant ν and K, substituting equation (3) into the continuity equation (1) gives

$$\nabla^2 p = 0 \tag{4}$$

for $x < \delta$. To examine solutions for the pressure field within the vapor layer in the limit $\varepsilon \to 0$, the coordinates

$$\xi = x/\varepsilon; \quad \eta = z$$

are introduced where the x-coordinate is stretched by the factor $1/\varepsilon$. In terms of these new coordinates, equation (4) becomes

$$\varepsilon^2 \frac{\partial^2 p}{\partial \eta^2} + \frac{\partial^2 p}{\partial \xi^2} = 0.$$

This form of the equation shows that for small ε derivatives along the vapor layer may be neglected with respect to those across the layer, and hence equation (4) may be written in the simple approximate form

$$\frac{\partial^2 p}{\partial x^2} = 0. \tag{5}$$

To obtain a solution, boundary conditions on $x = 0$ and $x = \delta$ must be prescribed. The vertical surface at $x = 0$ is taken to be impermeable so that the horizontal velocity must vanish and therefore $\partial p/\partial x = 0$ at $x = 0$. On the liquid–vapor boundary at $x = \delta$ the pressure is assumed to be $p = p_\Delta(z)$ where $p_\Delta(z)$ is to be determined from the flow outside the vapor layer. The solution of equation (5) satisfying these boundary conditions is simply

$$p = p_\Delta(z) \tag{6}$$

for $0 \leqslant x \leqslant \delta$. The vertical mass flux within the vapor layer is then given by

$$\rho u_z = -\frac{K}{\nu}\frac{\mathrm{d}p_\Delta}{\mathrm{d}z} \tag{7}$$

and the horizontal mass flux ρu_x, obtained by integrating the continuity equation (1) with respect to the x coordinate using (7), is given by

$$\rho u_x = -\frac{\partial}{\partial z}\int_0^x \frac{K}{\nu}\frac{\mathrm{d}p_\Delta}{\mathrm{d}z}\,\mathrm{d}x.$$

For constant K and ν, this reduces to

$$\rho u_x = -\frac{Kx}{\nu}\frac{\mathrm{d}^2 p_\Delta}{\mathrm{d}z^2}. \tag{8}$$

Therefore the pressure and mass flux vector within the vapor layer are determined solely by the pressure on the phase boundary.

The pressure variation on the phase boundary is determined by the flow outside the vapor layer in the region $x > \delta$. The liquid flowing in this region is assumed to be incompressible and to have a constant density ρ_L. Therefore equations (1) and (2) become

$$\nabla\cdot\bar{u} = 0 \tag{9}$$

$$\frac{\rho_L \nu_L}{K}\bar{u} = -\nabla p + \rho_L \bar{g}. \tag{10}$$

For flow outside the vapor layer, the boundary condition on $z = 0$ is $u_z = 0$ and on the edge of the vapor layer the mass flux into the vapor layer $\rho_L u_x|_{x=\delta}$ must balance the change with z of the mass flux within the vapor layer. For a thin vapor layer the boundary condition on $x = \delta$ may be applied on $x = 0$. With this approximation, which is commonly made in other applications of boundary-layer theory,

$$\rho_L u_x|_{x=0} = -\frac{\partial}{\partial z}\int_0^\delta \rho u_z\,\mathrm{d}x.$$

Using equation (7) this becomes

$$u_x|_{x=0} = \frac{K}{\rho_L \nu}\frac{\partial}{\partial z}\left(\delta \left.\frac{\partial p}{\partial z}\right|_{x=0}\right) \tag{11}$$

where $p|_{x=0}$ corresponds to $p_\Delta(z)$. Outside the vapor layer, the pressure is separated into two parts

$$\nabla p = \nabla p' + \rho_L \bar{g}$$

where $\nabla p'$ is the pressure gradient due to flow and $\rho_L \bar{g}$ is the hydrostatic pressure gradient. Then

$$\nabla p' = -\frac{\rho_L \nu_L}{K}\bar{u}. \tag{12}$$

For constant K and ν_L, substituting (12) into (9) gives

$$\nabla^2 p' = 0 \tag{13}$$

On $x = 0$ equation (11) becomes

$$u_x|_{x=0} = \frac{K}{\rho_L \nu}\left[\left(\left.\frac{\partial p'}{\partial z}\right|_{x=0} - \rho_L g\right)\frac{\mathrm{d}\delta}{\mathrm{d}z} + \delta \left.\frac{\partial^2 p'}{\partial z^2}\right|_{x=0}\right] \tag{14}$$

and on $z = 0$ since $u_z = 0$

$$\left.\frac{\partial p'}{\partial z}\right|_{z=0} = 0. \tag{15}$$

Equation (14) can be expressed solely in terms of the pressure by taking

$$\left.u_x\right|_{x=0} = -\frac{K}{\rho_L \nu_L}\left.\frac{\partial p'}{\partial x}\right|_{x=0}$$

from equation (12). Then the differential equation (13) along with boundary conditions (14) and (15) result in a mixed boundary value problem for the pressure field outside the vapor layer. This solution however depends on $\delta(z)$ which must be determined from the temperature field. The formulation of an energy equation governing the temperature field is discussed later.

A simple solution for flow outside the vapor layer can be obtained by assuming that u_z or equivalently that $\partial p'/\partial z$ is small. Equation (14) then becomes

$$\left.u_x\right|_{x=0} = -\frac{Kg}{\nu}\frac{\mathrm{d}\delta}{\mathrm{d}z}.$$

If u_z is small, the continuity equation reduces to $\partial u_x/\partial x = 0$ and therefore

$$u_x = -\frac{Kg}{\nu}\frac{\mathrm{d}\delta}{\mathrm{d}z}. \tag{16}$$

To obtain an explicit estimate of the error introduced by this approximation, note that the exact solution for u_x must satisfy $\nabla^2 u_x = 0$. For the solution given by equation (16) $\nabla^2 u_x = -(Kg/\nu)(\mathrm{d}^3\delta/\mathrm{d}z^3)$ so that the approximate solution is valid if $(Kg/\nu)(\mathrm{d}^3\delta/\mathrm{d}z^3)$ is small. For $\partial p'/\partial z$ small, $\mathrm{d}p_\Delta/\mathrm{d}z = -\rho_L g$ and the mass flux within the vapor layer given by equations (7) and (8) becomes

$$\rho u_z = \frac{K\rho_L g}{\nu} \tag{17}$$

$$\rho u_x = 0. \tag{18}$$

If u_z is small outside the vapor layer, liquid must move toward the heated vertical surface from large distances. A pressure gradient is required to generate the inflow so that the pressure p' must increase with increasing x. A difficulty therefore arises in specifying the pressure in the vicinity of the heated vertical surface. This behavior of the pressure also occurs in the single phase problem treated by Cheng and

Minkowycz [12], but in that problem it is not necessary to evaluate the pressure explicitly. Adjacent to a vertical surface in a permeable medium of finite size, a region of recirculating flow will be established as shown for example in the case of a cooling igneous intrusion by the finite difference models of Cathles [10]. Within such a region of recirculating flow, p' will be small compared to the hydrostatic pressure. Therefore, in situations of practical interest, it is reasonable to suppose that the pressure in the vicinity of the heated vertical surface will be hydrostatic to a first approximation.

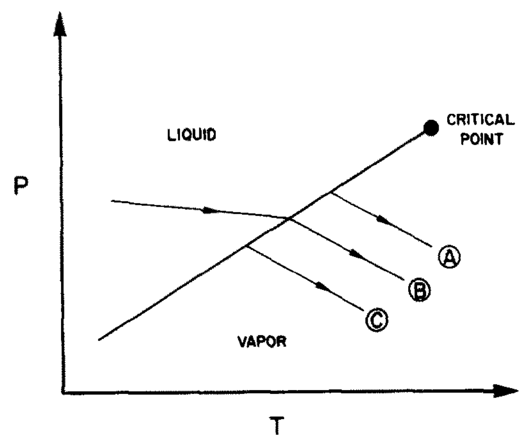

FIG. 2. Trajectories in pressure–temperature space described in the text.

It has so far been assumed that a phase boundary occurs across which liquid is transformed directly to vapor without the formation of a mixed phase region. Justification for this comes from examining the paths in pressure–temperature space followed by fluid moving along paths of motion such as shown in Fig. 1. As shown in Fig. 2, pressure–temperature paths begin within the liquid region above the vapor pressure curve. With pressure decreasing along the path, fluid moves with increasing temperature towards the vapor pressure curve. In crossing the vapor pressure curve three paths labelled A, B, and C are shown. In physical space a region of mixed phases could occur along any of the paths shown. Path A cannot occur unless pressure increases in the direction of flow which is inconsistent with Darcy's law. Path C cannot occur unless temperature decreases in the direction of flow, the negative x-direction. However, temperature must decrease with increasing x to transport heat from the heated surface to the region in which vaporization occurs. Neither of these physical inconsistencies occur along path B. Along path B if a mixed phase region were to form at the point where the vapor pressure curve is first intercepted, then the pressure would have to be uniform throughout the mixed phase region. As for path A this would be inconsistent with Darcy's law. Therefore fluid must follow a path such as B and must transform directly from liquid to vapor with no mixed phase region. These general arguments are consistent with the results of Rubin and Schweitzer [19] who treat the problem of one dimensional flow through a phase change.

Within the vapor layer, a boundary-layer form of the equation for conservation of thermal energy is applicable. Since $u_x \simeq 0$ and neglecting heat conduction compared with convection in the z direction, a balance exists between vertical convective heat transport and horizontal conductive heat transport which is expressed as

$$\rho c_p u_z \frac{\partial T}{\partial z} = k_m \frac{\partial^2 T}{\partial x^2} \tag{19}$$

where T is the temperature, c_p is the specific heat of the vapor assumed to be constant, and k_m is the thermal conductivity of the permeable medium. On $x = 0$, $T = T_0(z)$ where $T_0(z)$ is a prescribed temperature and on $x = \delta$, $T = T_\Delta$ where T_Δ is the vaporization temperature corresponding the hydrostatic pressure at a particular value of z.

Outside the vapor layer $u_z \simeq 0$ and again neglecting heat conduction in the z-direction gives a balance between horizontal conductive and convective heat transport which is expressed as

$$\rho_L c_{pL} u_x \frac{\partial T}{\partial x} = k_m \frac{\partial^2 T}{\partial x^2} \tag{20}$$

with $T \sim T_\infty$ for large x and $T = T_\Delta$ on $x = \delta$.

An additional boundary condition is required on $x = \delta$ which relates the rate at which heat is absorbed by vaporization to the net heat flux reaching the phase boundary. This heat balance can be expressed as

$$\rho H u_z \frac{\mathrm{d}\delta}{\mathrm{d}z} = -k_m \left.\frac{\partial T}{\partial x}\right|_{x=\delta^-} + k_m \left.\frac{\partial T}{\partial x}\right|_{x=\delta^+} \tag{21}$$

where H is the heat of vaporization per unit mass. The horizontal temperature gradient is discontinuous across the phase boundary and the notation $x = \delta^-$ and δ^+ refers to values approaching the phase boundary from $x < \delta$ and $x > \delta$ respectively.

Equation (20) may be solved without explicitly considering the variation of u_x or T_Δ with z. The solution which satisfies the boundary conditions given above is

$$T = T_\infty + (T_\Delta - T_\infty) \exp\left[\frac{\rho_L c_{pL} u_x}{k_m}(\delta - x)\right] \tag{22}$$

for $x \geqslant \delta$. Then differentiating and using equations (16) and (17)

$$k_m \left.\frac{\partial T}{\partial x}\right|_{x=\delta^+} = -\rho c_{pL}(T_\Delta - T_\infty) u_z \frac{\mathrm{d}\delta}{\mathrm{d}z}. \tag{23}$$

This may be used to simplify the form of equation (21) for the heat balance on the phase boundary to give

$$k_m \left.\frac{\partial T}{\partial x}\right|_{x=\delta^-} = -\rho u_z \bar{H} \frac{\mathrm{d}\delta}{\mathrm{d}z} \tag{24}$$

where $\bar{H} = H + c_{pL}(T_\Delta - T_\infty)$.

Using the previous result for ρu_z, equation (19) becomes

$$\frac{\partial T}{\partial z} = \left(\frac{\nu k_m}{\rho_L c_p g K}\right)\frac{\partial^2 T}{\partial x^2} \tag{25}$$

with $T = T_0(z)$ on $x = 0$ and $T = T_\Delta(z)$ on $x = \delta$. Equations (24) and (25) together with the boundary conditions on temperature form a Stefan problem.

AN EXACT SIMILARITY SOLUTION

Only one exact solution of the Stefan problem is known. This similarity solution can be applied to the present problem if T_0, T_Δ, and $\bar{H}$ do not vary with z. In this case the solution given by Carslaw and Jaeger [20] can be written in the form

$$T = T_0 + \frac{T_\Delta - T_0}{\operatorname{erf}\alpha}\operatorname{erf}\left(\frac{\alpha x}{\delta}\right) \tag{26}$$

with

$$\delta = 2\alpha(lz)^{1/2} \tag{27}$$

and

$$l = \left(\frac{\nu}{Kg}\right)\left(\frac{k_m}{\rho_L c_p}\right). \tag{28}$$

The constant α is determined from the equation

$$\frac{c_p(T_0 - T_\Delta)}{\bar{H}} = (\pi)^{1/2}\alpha\, e^{\alpha^2}\operatorname{erf}\alpha. \tag{29}$$

The value of α as a function of $c_p(T_0 - T_\Delta)/\bar{H}$ is plotted in Fig. 3. The heat flux from the vertical surface, calculated from equation (26), is given by

$$q = -k_m \frac{\partial T}{\partial x}\bigg|_{x=0} = \frac{2\alpha}{(\pi)^{1/2}\operatorname{erf}\alpha} k_m\left(\frac{T_0 - T_\Delta}{\delta}\right). \tag{30}$$

This can also be expressed in terms of a local Nusselt number $Nu_z = qz/k_m(T_0 - T_\infty)$ as

$$Nu_z = \frac{1}{(\pi)^{1/2}\operatorname{erf}\alpha}\left(\frac{T_0 - T_\Delta}{T_0 - T_\infty}\right)(Ra_z)^{1/2} \tag{31}$$

where

$$Ra_z = \frac{\rho_L c_p g K z}{\nu k_m} \tag{32}$$

is the local Rayleigh number. The local Nusselt number given by equation (31) has the same dependence on z as that derived from the similarity solution for a single phase liquid [12]. It depends, however, on a greater number of physical conditions and the local Rayleigh number is of a different, but equivalent, form. The form of the Rayleigh number for single phase flow can be obtained by multiplying the expression given in equation [32] by the product of the coefficient of thermal expansion of the liquid and a characteristic temperature difference. This product, which must be small if the Boussinesq approximation is to be applicable, represents a characteristic density difference normalized by a reference density. In the two phase problem, as treated in the present study, the expression $(\rho_L - \rho_V)/\rho_L$ is the appropriate normalized characteristic density difference, and for $\rho_V \ll \rho_L$ this is equal to unity.

To simplify the formulation of the two phase problem, the liquid outside the vapor layer has been assumed to be of constant density. However, the density of any real liquid will depend on temperature. The horizontal temperature variation outside the vapor layer as given by equation (22) will result in density variations that give rise to natural convection in the liquid phase. An equation to describe this flow can be obtained by cross-differentiating and subtracting the vector components of equation (10). With the boundary-layer approximation $\partial/\partial z \ll \partial/\partial x$ this gives

$$\frac{\mu_L}{K}\frac{\partial u_z}{\partial x} = -g\frac{\partial \rho_L}{\partial x}. \tag{33}$$

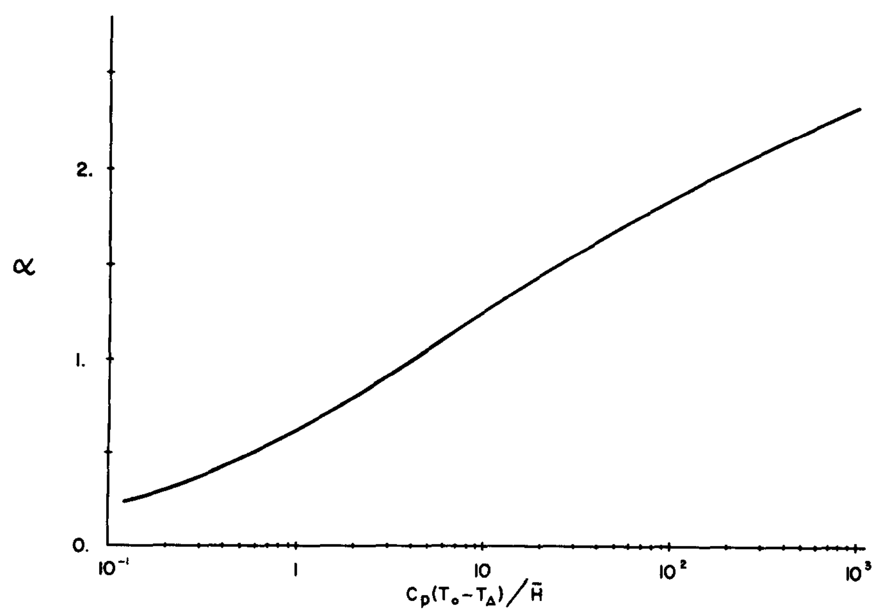

FIG. 3. The parameter α for the similarity solution.

The density of the liquid can be expressed as $\rho_L = \rho_\infty[1+\beta_L(T_\infty - T)]$ where ρ_∞ is the density of the liquid at temperature T_∞ and β_L is the coefficient of thermal expansion. Then requiring $u_z \sim 0$ as $x \to \infty$, equation (33) can be integrated to give

$$u_z = \frac{\beta_L g K}{\nu_L}(T - T_\infty). \quad (34)$$

Natural convection of liquid outside the vapor layer can be neglected if $|u_x| \gg |u_z|$. Using the expression for u_x in equation (16) results in the condition

$$\beta_L\left(\frac{\nu}{\nu_L}\right)(T_\Delta - T_\infty) \ll \frac{d\delta}{dz}$$

or for the similarity solution

$$\beta_L(T_\Delta - T_\infty)\left(\frac{\nu}{\nu_L}\right)\frac{1}{\alpha}(Ra_z)^{1/2} \ll 1. \quad (35)$$

For sufficiently large values of Ra_z and/or for small values of α (i.e. small values of $T_0 - T_\Delta$ or large values of $\bar{H}$, see Fig. 3), the condition [35] cannot be satisfied. A transition will occur from convection dominated by density differences due to the phase change to convection due to density variations within the liquid phase as treated by Cheng and Minkowycz [12].

APPLICATION TO PROBLEMS OF GEOLOGICAL INTEREST

For applications to cooling intrusions, the hydrostatic pressure and therefore the temperature T_Δ will vary significantly with height along the contact between an intrusion and surrounding rock. The heat of vaporization and the temperature at large distances from the contact will also vary. Therefore, the similarity solution is not strictly applicable to most cases of interest.

However, if T_0, T_Δ, and $\bar{H}$ change only slightly over a distance comparable to the thickness of the vapor layer, the approximation of local similarity may be applied. The basis for this approximation is to assume that the temperature distribution for the similarity solution, given by equation (26), applies locally and that the variation of the vapor layer thickness with height along the vertical surface may be determined by integrating the differential form of equation (27) obtained by assuming that α is locally constant

$$\frac{d\delta}{dz} = \alpha\left(\frac{l}{z}\right)^{1/2}. \quad (36)$$

In the non-similar case, α will vary slowly with z. It is clear from Fig. 3 that α varies slowly with $c_p(T_0 - T_\Delta)/\bar{H}$ suggesting that local similarity will be a good approximation for a wide range of conditions of practical interest. In the locally similar approximation, the heat flux from the vertical surface is determined from equation (30) using the local value of α and the vapor layer thickness determined from equation (36). The local similarity approximation has been applied to a variety of boundary-layer problems, generally with excellent results. For example, Minkowycz and Cheng [17] found it to be an excellent approximation for free convection about a vertical cylinder for a wide range of conditions. Their treatment, although more formally derived, is equivalent to the approach discussed above.

SUMMARY

Natural convection in a permeable medium due to the formation of a vapor layer adjacent to a vertical, heated surface has been treated using boundary-layer approximations for a thin vapor layer. In this situation liquid moving toward the heated surface transforms directly to vapor with no region of mixed phases. A similarity solution of the boundary-layer equations is presented for the case in which the temperature of the surface and of the liquid at large distances from the surface as well as thermodynamic properties such as the heat of vaporization do not vary with height along the surface. This leads to a simple analytical result for the rate of heat transfer from the surface. In obtaining these results it has been assumed that the density of the liquid does not vary with temperature. For a real liquid, the range of conditions for which this approximation is reasonable have been identified. It is suggested that realistic geologic problems of cooling igneous intrusions, which have been the primary motivation for the present study, can be treated using local similarity approximations.

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CONVECTION NATURELLE DIPHASIQUE ADJACENTE A UNE SURFACE VERTICALE CHAUDE DANS UN MILIEU PERMEABLE

Résumé—La convection naturelle diphasique adjacente à une surface verticale chaude, dans un milieu perméable, est traitée à partir des approximations de la couche limite avec la condition d'une couche mince de vapeur contre la surface chaude. Une solution de similarité est obtenue dans le cas où les différences de densité dues au changement de phase dominent celles dues aux variations de température dans la phase liquide hors de la couche de vapeur, et où la température de surface, la température au loin et la chaleur latente de vaporisation ne varient pas avec la distance le long de la surface. On discute de l'application de cette solution aux problèmes d'intérêt pratique utilisant l'approximation de la similarité locale.

FREIE KONVEKTION BEI ZWEI PHASEN IN EINEM PORÖSEN MEDIUM AN EINER BEHEIZTEN SENKRECHTEN WAND

Zusammenfassung—Die zweiphasige freie Konvektion in einem porösen Medium an einer beheizten senkrechten Wand wird mittels Approximationen der Grenzschicht für Fälle behandelt, bei denen die Dampfschicht an der beheizten Wand dünn ist. Eine Ähnlichkeitslösung wird für Fälle erhalten, bei denen die Dichteunterschiede infolge des Phasenwechsels diejenigen infolge Temperaturänderungen in der Flüssigkeitsphase außerhalb der Dampfschicht überwiegen und bei denen sich die Oberflächentemperatur sowie die Temperatur weit entfernt von der Wand und die Verdampfungswärme entlang der Wand nicht ändern. Die Anwendung dieser Lösung auf interessierende praktische Probleme wird unter Verwendung der näherungsweisen lokalen Ähnlichkeit diskutiert.

ДВУХФАЗНАЯ ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ ВБЛИЗИ ВЕРТИКАЛЬНОЙ НАГРЕВАЕМОЙ ПОВЕРХНОСТИ В ПРОНИЦАЕМОЙ СРЕДЕ

Аннотация — С помощью приближений пограничного слоя исследуется двухфазная естественная конвекция в проницаемой среде вблизи нагреваемой вертикальной поверхности при наличии на поверхности тонкого слоя пара. Методом подобия получено решение для случая, когда разность плотностей, обусловленная фазовым переходом превалирует над разностью, вызываемой изменениями температуры в жидкой фазе за пределами парового слоя, и когда температура поверхности, температура на некотором расстоянии от поверхности и теплота парообразования не изменяются с расстоянием вдоль пластины. Обсуждается возможность применения данного решения к задачам, представляющим практический интерес.